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Calculation of laminar film condensation in/on inclined elliptical tubes

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Abstract—The local heat transfer coefficient for laminar gravity controlled Nusselt type film condensation is calculated analytically. The area average mean value is calculated by numerical integration of the local value over the condensation surface or, alternatively, by integrating the condensate flow rate along the border of the heat transfer area. The results for the elliptical tube of finite and infinite length are compared with those obtained for an equivalent circular tube of equal surface. The elliptical deformation (flattened at the side) increases the heat transfer coefficient, especially with short tubes.

1. INTRODUCTION

CONDENSATION inside elliptical tubes is frequently applied in air cooled power station condensers, and condensation heat transfer in such inclined tubes is subject to recent experimental research [1].

The estimation of heat transfer coefficients using known calculation methods is unreliable and special calculation procedures or formulas have still to be developed.

As a first step in this direction the Nusselt type laminar film condensation [2] in or on inclined elliptical tubes is investigated analytically in this paper. Hassan and Jacob [3] calculated the special case of a circular cylinder and their numerical results compare well with the more general (closed form) solution, presented in this paper.

The same assumptions are made as by Hassan and Jacob. The additional minor effect of surface tension on film flow, which occurs mainly in non-circular tubes, is neglected here.

2. DERIVATION OF THE DIFFERENTIAL EQUATION FOR THE FILM THICKNESS

An inclined elliptical tube of finite length and uniform wall surface temperature is considered. One axis of the cross-sectional ellipse is horizontal.

Consider an incremental plate of width dx and length dU lying on the tube at a radial angle β, whereas the tube is inclined from the horizontal plane at an angle ψ (Fig. 1). One side of the plane (tube surface), in the positive y direction, is exposed to saturated vapour at uniform temperature T_S, that is greater than the surface temperature T_W.

Because of the inclination, the condensate film is simultaneously drained to the x and U directions, respectively.

$$\frac{\partial^2 w_x}{\partial y^2} = -\frac{1}{\eta}\rho g \sin\psi \tag{1a}$$

$$\frac{\partial^2 w_U}{\partial y^2} = -\frac{1}{\eta}\rho g \cos\psi \sin\gamma. \tag{1b}$$

Integrating equations (1) with respect to y and utilizing the classical boundary conditions of no slip at the solid interface yields:

$$\bar{w}_x = \frac{\rho g}{3\eta}\sin\psi\,\delta^2 \tag{2a}$$

$$\bar{w}_U = \frac{\rho g}{3\eta}\cos\psi\sin\gamma\,\delta^2. \tag{2b}$$

The heat conducted through the film to the incremental area $dx\cdot dU$ causes a change of condensate flow (Fig. 2)

$$\frac{\lambda}{\delta}\,dx\,dU\Delta T = r\left(\frac{\partial}{\partial x}(\bar{w}_x\rho\delta\,dU)\,dx + \frac{\partial}{\partial U}(\bar{w}_U\rho\delta\,dx)\,dU\right) \tag{3}$$

where the incremental periphery (Fig. 3)

$$dU = a\sqrt{(1-\varepsilon^2\sin^2\phi)}\,d\phi \tag{4}$$

and the eccentricity

$$\varepsilon^2 = (a^2-b^2)/a^2. \tag{5}$$

Combining equations (2)–(5) yields the quasilinear partial differential equation

NOMENCLATURE

a, b	semi axis of ellipse	β	angle
c_1, c_2	integration constants	γ	angle
g	gravitational acceleration	δ	film thickness
L	partial circumference	Δ	finite difference
$\dot{m}$	mass flux	ε	eccentricity
r	latent heat of condensation	η	condensate viscosity
R	radius	λ	thermal conductivity of condensate
t	dummy variable for ϕ	ρ	density of condensate
T	temperature	ϕ	eccentric angle
U	coordinate in circumferential direction, circumference	ψ	angle of inclination.
v	dummy variable for X	Subscripts	
w	velocity	EL	of the actual ellipse
x	coordinate, tube length	KREIS	of the equivalent circular tube
X	dimensionless tube length	L	for the partial circumference
y	coordinate	S	at saturation conditions
Z	dimensionless film thickness.	W	at wall surface
		x	in x-direction
Greek symbols		U	in U-direction
α	heat transfer coefficient	1/2	one half of.

$$\frac{\partial Z}{\partial X}+\frac{\sin\phi}{1-\varepsilon^2\sin^2\phi}\frac{\partial Z}{\partial\phi}=\frac{4}{3}\left(1-\frac{\cos\phi}{(1-\varepsilon^2\sin^2\phi)^2}Z\right) \tag{6}$$

for the local dimensionless condensate film thickness

$$Z=\frac{\delta^4}{\dfrac{3\eta\Delta T\lambda}{\rho^2 gr\cos\psi}\left(\dfrac{a^2}{b}\right)} \tag{7}$$

as function of the dimensionless tube length

$$X=\frac{x}{\left(\dfrac{a^2}{b}\right)\tan\psi} \tag{8}$$

and the angle ϕ.

The boundary conditions are

$$\phi=0 \quad \text{or} \quad \phi=\pi \qquad \frac{\partial Z}{\partial\phi}=0 \tag{9a}$$

$$x=0 \qquad Z=0 \tag{9b}$$

$$x=\infty \qquad \frac{\partial Z}{\partial X}=0. \tag{9c}$$

3. SOLUTION AND CALCULATION OF THE HEAT TRANSFER COEFFICIENT

3.1. *Infinite tube length*

First the simple limiting case of an infinite length $X=\infty$ is considered.

At the end of this tube the film flow is fully

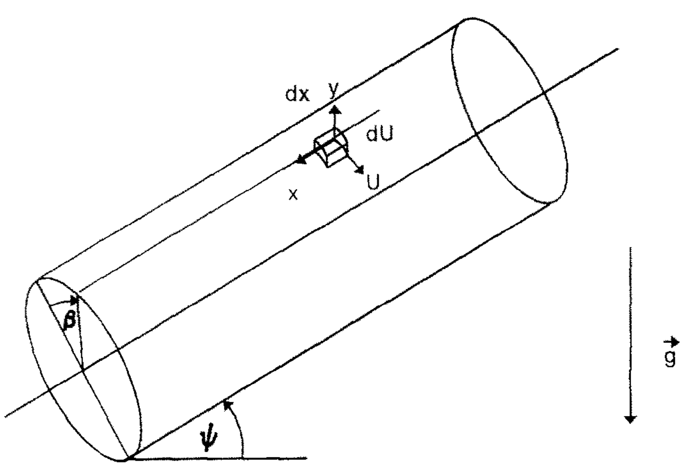

FIG. 1. Inclined elliptical tube in the gravity field.

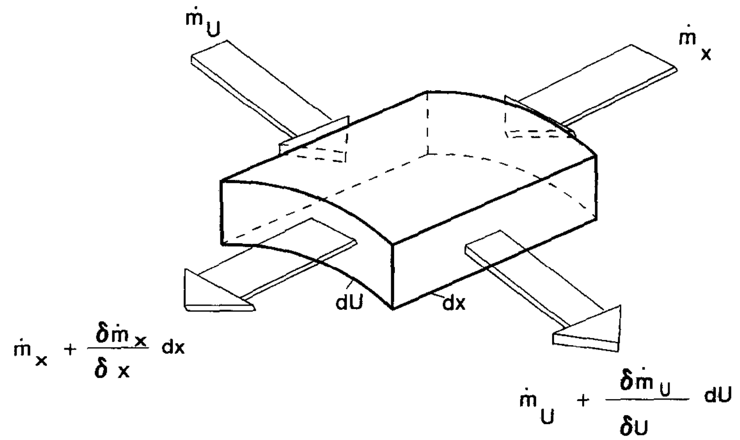

FIG. 2. Elementary volume of condensate film.

developed and the local film thickness Z is only a function of ϕ.

Thus the boundary condition (9c) reduces equation (6) to the ordinary linear inhomogeneous differential equation

$$\frac{dZ}{d\phi}+\frac{4}{3}\frac{Z}{(1-\varepsilon^2\sin^2\phi)\tan\phi}-\frac{4}{3}\frac{1-\varepsilon^2\sin^2\phi}{\sin\phi}=0. \tag{10}$$

In the special case of a circular tube of radius $R = a = b$ this equation simplifies to the well known equation of Nusselt [2].

$$\frac{dZ}{d\phi}+\frac{4}{3}\frac{Z}{\tan\phi}-\frac{4}{3}\frac{1}{\sin\phi}=0. \tag{11}$$

In the more general case of an elliptical inclined tube of infinite length one finds from equation (10) and the boundary condition $\phi = 0 \rightarrow (dZ/d\phi) = 0$ the analytical solution

$$Z=\frac{4}{3}\left(\frac{\sqrt{(1-\varepsilon^2\sin^2\phi)}}{\sin\phi}\right)^{4/3}\int_0^{\phi}\sin^{1/3}t \times(1-\varepsilon^2\sin^2 t)^{1/3}\,dt. \tag{12}$$

From the local film thickness

$$\delta=\left(\frac{3\eta\Delta T\lambda}{\rho^2 gr\cos\psi}\left(\frac{a^2}{b}\right)Z\right)^{1/4} \tag{13}$$

the local heat transfer coefficient

$$\alpha=\frac{\lambda}{\delta} \tag{14}$$

can be calculated, assuming a linear temperature profile in the film.

Integrating along the periphery U ($U_{1/2}$ = one half of the total circumference) yields the area average heat transfer coefficient

$$\bar{\alpha}_L=\frac{1}{L}\int_0^L\alpha(U)\,dU \quad 0\leqslant L\leqslant U_{1/2} \tag{15}$$

of an inclined elliptical tube of infinite length.

3.2. *Finite tube length*

In the general case of finite tube length equation (6) has to be solved taking the boundary conditions according to equations (9) into account.

An analytical solution is possible applying the method of characteristics. The subsidiary equations read:

$$dX=\frac{d\phi}{\dfrac{\sin\phi}{1-\varepsilon^2\sin^2\phi}}=\frac{dZ}{\dfrac{4}{3}\left(1-\dfrac{\cos\phi}{(1-\varepsilon^2\sin^2\phi)^2}Z\right)}. \tag{16}$$

Rearranging and integrating yields

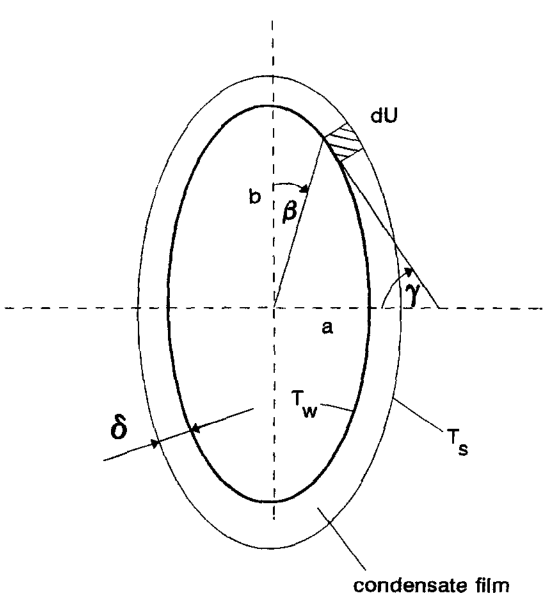

FIG. 3. Cross-section of tube with condensate film.

$$X = \varepsilon^2 \cos\phi + \ln[c_1 \tan(\phi/2)] \tag{17}$$

and

$$Z = \frac{4}{3}\left(\frac{\sqrt{(1-\varepsilon^2\sin^2\phi)}}{\sin\phi}\right)^{4/3} \times \left(\int_0^\phi \sin^{1/3} t(1-\varepsilon^2\sin^2 t)^{1/3}\,dt + c_2\right). \tag{18}$$

Equation (17) describes the flow path of the condensate on the condensation surface.

Incorporating in equations (17) and (18) the boundary condition according to equation (9b) leads finally to

$$Z = \frac{4}{3}\left(\frac{\sqrt{(1-\varepsilon^2\sin^2\phi)}}{\sin\phi}\right)^{4/3} \times \left(\int_0^\phi \sin^{1/3} t(1-\varepsilon^2\sin^2 t)^{1/3}\,dt - \int_0^{\phi^*} \sin^{1/3} t(1-\varepsilon^2\sin^2 t)^{1/3}\,dt\right). \tag{19}$$

Comparing with equation (12) reveals that the second integral in equation (19) takes the finite tube length into account.

The value of ϕ^* has to be determined from

$$\frac{\exp(-\varepsilon^2\cos\phi^*)}{\tan\left(\frac{\phi^*}{2}\right)} = \frac{\exp(X-\varepsilon^2\cos\phi)}{\tan\left(\frac{\phi}{2}\right)}. \tag{20}$$

At the top ($\phi = 0$) and the bottom ($\phi = \pi$) of the ellipse equation (6) together with the boundary condition (9a) gives

$$Z(\phi = 0) = 1-\exp\left(-\frac{4}{3}X\right) \tag{21}$$

$$Z(\phi = \pi) = \exp\left(\frac{4}{3}X\right)-1. \tag{22}$$

In the special case of a circular cylinder of radius $R = a = b$ equation (6) turns to

$$\frac{\partial Z}{\partial X} + \sin\phi\frac{\partial Z}{\partial \phi} = \frac{4}{3}(1-Z\cos\phi). \tag{23}$$

Equation (23) was also derived and solved using a finite difference method by Hassan and Jacob [3]. The above derived analytical solution according to equations (19) and (20) now simplifies to

$$Z = \frac{4}{3}\frac{1}{\sin^{4/3}\phi}\left(\int_0^\phi \sin^{1/3} t\,dt - \int_0^{\phi^*} \sin^{1/3} t\,dt\right) \tag{24}$$

and

$$\phi^* = 2\arctan\left[\tan\left(\frac{\phi}{2}\right)\Big/\exp(X)\right]. \tag{25}$$

Thus, in the most general case of an inclined elliptical tube of finite length, the local heat transfer coefficient can be calculated using equations (19)–(22) and (13), (14).

The area average heat transfer coefficient can be found by the integration

$$\bar{\alpha} = \frac{\lambda}{XU_{1/2}}\left[\frac{3\eta\Delta T\lambda}{\rho^2 gr\cos\psi}\left(\frac{a^2}{b}\right)\right]^{-1/4}\int_0^X\int_0^L \frac{dU\,dv}{\sqrt[4]{(Z)}}. \tag{26}$$

Alternatively, this mean value can be calculated applying the energy balance for the heat transfer area (see equation (8))

$$Lx = LX\left(\frac{a^2}{b}\right)\tan\psi. \tag{27}$$

The heat transferred to this area decreases the enthalpy of the condensing vapour

$$\bar{\alpha}Lx\Delta T = r(\dot{m}_{L=\text{const.}} + \dot{m}_{X=\text{const.}}). \tag{28}$$

The mass flow rates $\dot{m}_{L=\text{const.}}$ and $\dot{m}_{X=\text{const.}}$ are the condensate streams crossing the borders $L = \text{const.}$ and $X = \text{const.}$ of condensation surface Lx, respectively.

Applying equations (19), (20) and (2a), (2b) yields

$$\dot{m}_{L=\text{const.}} = \left(\frac{\rho^2 g}{3\eta}\right)^{1/4}\left[\frac{\Delta T\lambda}{r\cos\psi}\left(\frac{a^2}{b}\right)\right]^{3/4} \times \sin\psi\left(\frac{a^2}{b}\right)\sin\gamma\int_0^X Z_{L=\text{const.}}^{3/4}\,dv \tag{29}$$

and

$$\dot{m}_{X=\text{const.}} = \left(\frac{\rho^2 g}{3\eta}\right)^{1/4}\left[\frac{\Delta T\lambda}{r\cos\psi}\left(\frac{a^2}{b}\right)\right]^{3/4} \times \sin\psi\int_0^L Z_{X=\text{const.}}^{3/4}\,dU. \tag{30}$$

The numerical evaluation of equation (29) causes problems because for $L = U_{1/2}$ (or $\phi = \pi$) and $X \to \infty$ the local film thickness $Z \to \infty$ (equation (22)).

This problem can be overcome by substituting $L = U_{1/2}$ or $\phi = \pi$ by values very close to this lowest point of the tube. Numerical calculations show that with $\phi = 3.124$ sufficient accuracy can be expected. The relative errors of $\bar{\alpha}$ are then usually below 2%.

4. RESULTS AND DISCUSSION

For the illustration of the numerical results a dimensionless coordinate in U-direction ($L/U_{1/2}$) and a heat transfer ratio $(\bar{\alpha}_{EL}/\bar{\alpha}_{KREIS})_L$ are introduced. The heat transfer ratio is the ratio of two heat flow rates or two mean heat transfer coefficients. The subscript 'EL' refers to the actual elliptical tube. The subscript

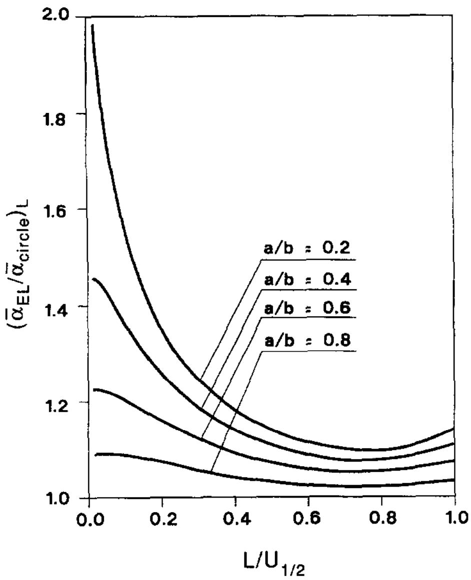

FIG. 4. Heat transfer ratio as function of the dimensionless coordinate $L/U_{1/2}$ for various values of $a/b < 1$.

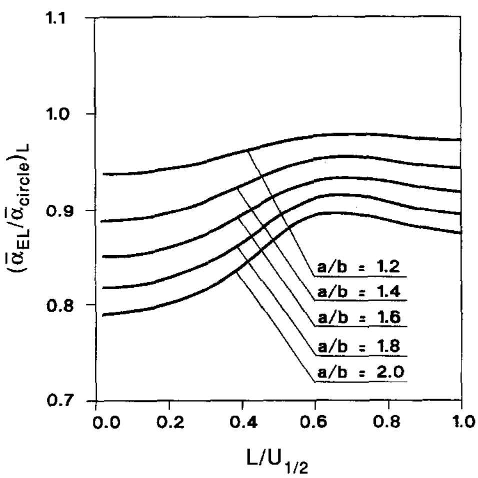

FIG. 5. Heat transfer ratio as function of the dimensionless coordinate $L/U_{1/2}$ for various values of $a/b > 1$.

'(KREIS)' refers to an equivalent circular tube with the same condensation surface area or circumference. The heat transfer ratio is a criterion for the utility of an elliptical deformation of a circular tube.

First the tube of infinite length is considered.

Figures 4 and 5 show the heat transfer ratio as function of the dimensionless coordinate $L/U_{1/2}$ (in this case dimensionless flow path of the condensate) for various values of a/b.

The decisive values $(\bar{\alpha}_{EL}/\bar{\alpha}_{KREIS})_L$ at $L/U_{1/2} = 1$, which are ratios of the total area average heat transfer coefficients or the total heat fluxes, respectively, are plotted in Fig. 6 as function of the ratio a/b. In the limiting case $a/b \to \infty$ the calculation is no longer valid, because a condensate flow parallel to the wall surface is assumed which does not occur on a horizontal plane. Figure 6 shows that for $a/b < 1$ the total heat transfer ratio becomes >1 and vice versa. An elliptical deformation of a circular tube improves heat transfer only if $a/b < 1$. For that reason ellipses with $a/b > 1$ are not considered any more.

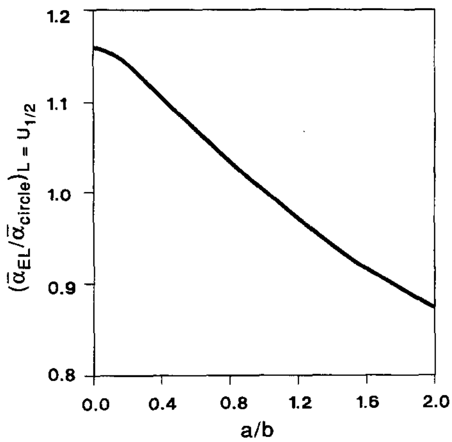

FIG. 6. Total heat transfer ratio $(\bar{\alpha}_{EL}/\bar{\alpha}_{KREIS})_{L=U_{1/2}}$ as function of axis ratio a/b.

In the limiting case $a/b \to 0$ the ellipse turns to a vertical plate with the maximum total heat transfer ratio 1.157.

A more significant increase of heat transfer is obtained with inclined elliptical tubes of finite length, as shown in Figs. 7 and 8.

In Fig. 7 the total heat transfer ratio is plotted as function of dimensionless tube length X for various values of $a/b < 1$. For $X \to \infty$ the heat transfer ratios assume the minimum values shown in Fig. 6.

The effect of tube length is even better demonstrated in Fig. 8, where inclined elliptical tubes of finite and infinite length are compared.

5. CONCLUSIONS

1. The local heat transfer coefficient for Nusselt type laminar film condensation in/on inclined elliptical tubes can be calculated analytically (equations (13), (14), (19), (20)).

2. A comparison (Figs. 6 and 7) with an equivalent circular tube of equal condensation surface area clearly demonstrates the superiority of elliptical tubes (flattened at the side).

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Note added in proof—After acceptance of this paper the paper 'Role of surface tension and ellipticity in laminar film condensation on a horizontal elliptical tube' (Yang and Chen, *Int. J. Heat Mass Transfer* **36**, 3141–3135 (1993)) has been published. It represents an interesting special case of our results for a horizontal elliptical tube. In this respect the paper *Fortschr.-Ber. VDI* **19**(7), VDI-Verlag (1986) from G. Fieg should be mentioned in which the calculation of laminar film condensation in/on inclined elliptical tubes (for infinite and finite tubes) under the effects of gravity *and* surface tension has been already considered.

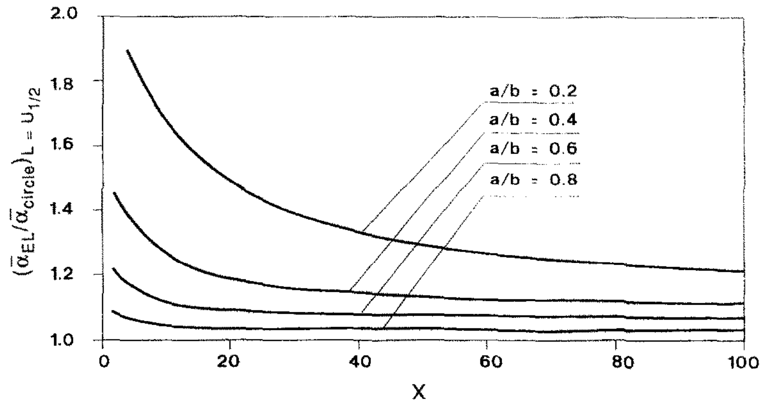

FIG. 7. Total heat transfer ratio as function of dimensionless tube length X.

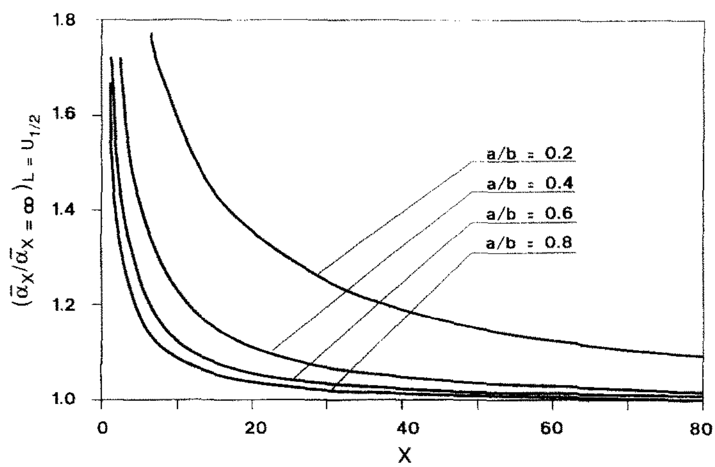

FIG. 8. Ratio of heat flux or heat transfer coefficients for finite and infinite elliptical tubes as function of dimensionless tube length for various axis ratios a/b.

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